Limits and Derivatives

Assertion Reason Questions

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of thefollowing choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

1.

Assertion (A): $\lim_{x \to 1} \frac{ax^2+bx+c}{cx^2+bx+a}$ is equal to 1, where $a+b+c \neq 0$.

Reason (R): $\lim_{x \to -2} \frac{\frac{1}{x}+\frac{1}{2}}{x+2}$ is equal to $\frac{1}{4}$.

Ans. (c) (A) is true but (R) is false.

Explanation: Given, $\lim_{x \to 1} \frac{ax^2+bx+c}{cx^2+bx+a}$

$$= \frac{a \times (1)^2 + b \times 1 + c}{c \times (1)^2 + b \times 1 + a}$$

$$= \frac{a+b+c}{c+b+a} = 1$$

Given, $\lim_{x \to -2} \frac{\frac{1}{x}+\frac{1}{2}}{x+2}$

$$= \lim_{x \to -2} \frac{(2+x)}{2x(x+2)} = \lim_{x \to -2} \frac{1}{2x}$$

$$= \frac{1}{2(-2)} = -\frac{1}{4}$$

2.

Assertion (A): $\lim_{x \to 0} \frac{\sin ax}{bx}$ **is equal to** $\frac{a}{b}$.

Reason (R): $\lim_{x \to 0} \frac{\sin x}{x} = 1$.

Ans. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

Explanation: Given, $\lim_{x \to 0} \frac{\sin ax}{bx} = \lim_{x \to 0} \frac{(a)\sin ax}{b(ax)}$

[dividing and multiplying by a]

$$= \frac{a}{b} \times 1 = \frac{a}{b} \left[\because \lim_{x \to 0} \frac{\sin ax}{ax} = 1 \right]$$

3.

Assertion (A): $\lim_{x \to 0} \frac{\sin ax + bx}{ax + \sin bx}$ **is equal to –2.**

Reason (R): $\lim_{x \to 1} (5x^3 + 5x + 1)$ **is equal to 11.**

Ans. (d) (A) is false but (R) is true.

Explanation: $\lim_{x \to 0} \frac{\sin ax + bx}{ax + \sin bx}$

Dividing each term by x, we get

$$= \lim_{x \to 0} \frac{\frac{\sin ax}{x} + \frac{bx}{x}}{\frac{ax}{x} + \frac{\sin bx}{x}} = \lim_{x \to 0} \frac{\frac{a \sin ax}{ax} + b}{a + \frac{b \sin bx}{bx}}$$

$$= \frac{a \times 1 + b}{a + b \times 1} = \frac{a+b}{a+b} = 1 \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \right]$$

Given, $\lim_{x \to 1} (5x^3 + 5x + 1)$

$$= 5(1)^3 + 5(1) + 1 = 5 + 5 + 1 = 11$$

4.

Assertion (A): $\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$ is equal to π.

Reason (R): $\lim_{x \to 0} \frac{\cos x}{\pi - x}$ is equal to $\frac{1}{\pi}$.

Ans. (d) (A) is false but (R) is true.

Explanation: Given, $\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$

Let, $\pi - x = h$, As $x \to \pi$, then $h \to 0$,

$$\therefore \lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)} = \lim_{h \to 0} \frac{\sin h}{\pi h}$$

$$= \lim_{h \to 0} \frac{1}{\pi} \times \frac{\sin h}{h}$$

$$= \frac{1}{\pi} \times 1 = \frac{1}{\pi} \left[\because \lim_{h \to 0} \frac{\sin h}{h} = 1\right]$$

Given, $\lim_{x \to 0} \frac{\cos x}{\pi - x}$

Putting, the limit directly, we get

$$\frac{\cos 0}{\pi - 0} = \frac{1}{\pi}$$

5. Let u = f(x) and v = g(x). Then,

Assertion(A): (uv)'=u'v+ uv' is a Leibnitz rule or product rule.

Reason (R): $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v'}$ is a Leibnitz rule or quotient rule.

Ans. (c) (A) is true but (R) is false.

Explanation: Let u = f(x) and v = g(x).

Then, (uv)'=u'v + uv'

This is referred as Leibnitz rule or the product rule for differentiating product of functions.

Similarly, the quotient rule is $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$.

6. Assertion (A): The derivation of f(x) = x^3 is x^2.

Reason (R): The derivation of f(x) = x^n is nx^{n-1}

Ans. (d) A is false but R is true.

Explanation: We have, f(x) = x3

We know that the derivation of x^n is nx^{n-1}

= f'(x) = $3x^2x^2$

7.

Assertion (A): The derivation of $y = 2x - \frac{3}{4}$

is 2.

Reason (R): The derivation of y = cx is c.

Ans. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

Explanation: We have, $y = 2x - \frac{3}{4}$

$$\Rightarrow \frac{dy}{dx} = (2 \times 1) - 0 = 2$$

8. Assertion (A): The derivation of

$$h(x) = \frac{x + \cos x}{\tan x} \quad \text{is}$$

$$\frac{(1 - \sin x)\tan x - (x + \cos x)\sec^2 x}{(\tan x)^2}.$$

Reason (R): $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{(v)^2}$.

Ans. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

Explanation: We have, $h(x) = \frac{x + \cos x}{\tan x}$ ...(i)

Differentiating both sides of (i) w.r.t. 'x, we get

$$h'(x) = \frac{(x+\cos x)' \tan x - (x+\cos x)(\tan x)'}{(\tan x)^2}$$

$$= \frac{(1-\sin x)\tan x - (x+\cos x)\sec^2 x}{(\tan x)^2}$$